THE CHINESE UNIVERSITY OF HONG KONG

DEPARTMENT OF MATHEMATICS

MATH1010H/I/J University Mathematics 2017-2018

Assignment 5

Due Date: 22 Mar 2018 (Thursday)

1. Let $f(x) = \dfrac{|x|(x + 16)}{x - 2}$ for $x \neq 2$.

   (a) (i) Is $f(x)$ differentiable at $x = 0$? Why?

   　　(ii) Find $f'(x)$ and $f''(x)$ for $x \neq 0$.

   (b) Solve

   　　(i) $f'(x) > 0$ and $f'(x) < 0$;

   　　(ii) $f''(x) > 0$ and $f''(x) < 0$.

   (c) Find the relative extreme point(s) and point(s) of inflection of the graph $y = f(x)$.

   (d) Find all asymptote(s) of the graph $y = f(x)$.

   (e) Sketch the graph of $y = f(x)$.

2. Evaluate the following limits.

   (a) $\displaystyle\lim_{x \to 0} \dfrac{e^{2x} - 1 - 2x}{x^2}$

   (b) $\displaystyle\lim_{x \to 0} \dfrac{\tan^{-1} x}{2x}$

   (c) $\displaystyle\lim_{x \to 0^+} x(\ln x)^2$

   (d) $\displaystyle\lim_{x \to 1} \left(\dfrac{1}{\ln x} - \dfrac{x}{x - 1} \right)$

   (e) $\displaystyle\lim_{x \to +\infty} \left(\dfrac{\sin^2 x}{x} \right)^{\frac{1}{x^2}}$

   (f) $\displaystyle\lim_{x \to 1} x^{\frac{1}{1-x}}$

3. Find the Taylor polynomial of degree 3 of the following functions at $x = 0$.

   (a) $f(x) = e^{\cos 2x}$

   (b) $f(x) = e^{2x} \ln(1 - x)$

   (c) $f(x) = \sec x$

4. Find the Taylor series of the following functions at $x = 0$.

   (a) $f(x) = \dfrac{4}{2 - x^2}$

   (b) $f(x) = \sqrt{1 + x}$

   (c) $f(x) = \ln(4 + 3x)$

   (d) $f(x) = \dfrac{2x + 3}{(x + 1)(x + 3)}$ (Hint: Resolve it into partial fractions first.)

5. Let $f(x) = \dfrac{1}{1-x}$.

   By considering $f'(x)$, $f''(x)$, find the Taylor series generated by $\dfrac{1}{(1-x)^2}$ and $\dfrac{1}{(1-x)^3}$ at $x = 0$.

6. By considering the Taylor series of $\sin(x^2) - x\sin x$, find $\displaystyle\lim_{x\to 0} \dfrac{\sin(x^2) - x\sin x}{x^4}$.

7. Let $f(x) = (1 + x^2)\sin(x^2)$

   (a) Find the Taylor series generated by $f(x)$ at $x = 0$.

   (b) Hence, find $f^{(100)}(0)$ and $f^{(101)}(0)$.